# N(A)-Level Additional Mathematics 4051

## Paper 1

**Tuesday 6 October 2026 · 1 hour 45 minutes · 70 marks**

**Suggested worked answers by Math Vigilante. Unofficial.**

### How to read this document

Each question is referred to by its number and topic. The questions themselves are not reproduced: read this beside your copy of the paper.

The final answer to each part is in red.

M1, A1 and B1 mark where each mark is earned, in our suggested allocation; the official mark scheme may differ.

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**TOPICS TESTED**

- Q1: Completing the square
- Q2: Long division
- Q3: Area: curve and line
- Q4: Exact trig ratios
- Q5: Trig equation
- Q6: Curve from its gradient
- Q7: Trig identity, hence solve
- Q8: Maximum volume
- Q9: Partial fractions
- Q10: Definite integral
- Q11: Trig graph
- Q12: Quadratic inequalities
- Q13: Quadratic in context

*Our worked solution to every question, by number. The questions themselves are not reproduced: read this beside your copy of the paper. 70 marks, 13 questions. Marks for Q5 and Q7(a) are estimated: their mark brackets were not legible in the copy we worked from, and they total 7 (the paper is out of 70).*

This document consists of **8** printed pages.

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**1** **[Completing the square; decreasing function]**

**(a)** [2]

*Marking codes at the step that earns each mark: M1 = method, A1 = accuracy (needs the M mark before it), B1 = independent mark.*

$$\mathrm{f}(x) = -3(x^2 - 3x) - 10$$
$$= -3\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] - 10 \qquad \textbf{[M1]}$$
$$= -3\left(x - \frac{3}{2}\right)^2 + \frac{27}{4} - 10$$
$$= \boldsymbol{-3\left(x - \frac{3}{2}\right)^2 - \frac{13}{4}} \qquad \textbf{[A1]}$$

**(b)** [1]

**Since $\left(x - \frac{3}{2}\right)^2 \geq 0$, $-3\left(x - \frac{3}{2}\right)^2 \leq 0$, so $\mathrm{f}(x) \leq -\frac{13}{4} < 0$ for all $x$.** **[B1]**

**(c)** [1]

The graph is an upside-down parabola with its maximum at $x = \frac{3}{2}$.

Or: $\mathrm{f}'(x) = 9 - 6x < 0$ when $x > \frac{3}{2}$.

$\boldsymbol{x > \frac{3}{2}}$ **[B1]**

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**2** **[Polynomials: long division]**

[4]

$$2x(x^2 - x + 6) = 2x^3 - 2x^2 + 12x \qquad \textbf{[M1]}$$
$$(2x^3 - 2x^2 + 5x + 7) - (2x^3 - 2x^2 + 12x) = -7x + 7$$

$$\boldsymbol{\frac{2x^3 - 2x^2 + 5x + 7}{x^2 - x + 6} = 2x + \frac{-7x + 7}{x^2 - x + 6}}$$

$\boldsymbol{a = 2}$ **[A1]** $\boldsymbol{b = -7}$ **[A1]** $\boldsymbol{c = 7}$ **[A1]**

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**3** **[Curve and line: intersections, area]**

**(a)** [1]

**Between A and B the curve is above the line, but between B and C the line is above the curve. One integral of (curve − line) from A to C would subtract one region from the other, so the area must be split at B.** **[B1]**

**(b)** [3]

$2x^3 - 4x^2 + 9 = 6x + 9$ **[M1]**

$2x^3 - 4x^2 - 6x = 0$

$2x(x^2 - 2x - 3) = 0$

$2x(x + 1)(x - 3) = 0$ **[M1]**

$\boldsymbol{x_A = -1,\ \ x_B = 0,\ \ x_C = 3}$ **[A1]**

**3** *(continued)*

**(c)** [3]

$$\int_{-1}^{0}(2x^3 - 4x^2 - 6x)dx = \left[\frac{x^4}{2} - \frac{4x^3}{3} - 3x^2\right]_{-1}^{0} \quad \textbf{[M1]}$$

$$= 0 - \left(\frac{1}{2} + \frac{4}{3} - 3\right)$$

$$= \frac{7}{6}$$

$$\int_{0}^{3}(-2x^3 + 4x^2 + 6x)dx = \left[-\frac{x^4}{2} + \frac{4x^3}{3} + 3x^2\right]_{0}^{3} \quad \textbf{[M1]}$$

$$= -\frac{81}{2} + 36 + 27$$

$$= \frac{45}{2}$$

$$\text{Total area} = \frac{7}{6} + \frac{45}{2}$$

$$= \boldsymbol{\frac{71}{3} = 23\frac{2}{3}\ \text{units}^2} \quad \textbf{[A1]}$$

*[Note: Integrate (upper − lower) on each part, so both pieces come out positive.]*

**4** **[Exact trig ratios (no calculator)]**

**(a)** [3]

$$\tan\theta = \frac{1}{2} \qquad \textbf{[B1]}$$

$$\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{4} = \frac{5}{4} \qquad \textbf{[M1]}$$

$$\boldsymbol{\sec\theta = \frac{\sqrt{5}}{2}} \qquad \textbf{[A1]}$$

*[Note: θ is acute, so sec θ is positive: take the positive root.]*

**(b)** [2]

$$\tan(\theta + 45°) = \frac{\tan\theta + \tan 45°}{1 - \tan\theta\tan 45°} \qquad \textbf{[M1]}$$

$$= \frac{\frac{1}{2} + 1}{1 - \frac{1}{2}}$$

$$= \mathbf{3} \qquad \textbf{[A1]}$$

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**5** **[Trig equation]**

[4]

$$\frac{\sin\theta}{\cos\theta} + 2\sin\theta = 0$$

$$\sin\theta + 2\sin\theta\cos\theta = 0$$

$$\sin\theta(1 + 2\cos\theta) = 0 \qquad \textbf{[M1]}$$

$$\boldsymbol{\sin\theta = 0:\ \theta = 0°, 180°, 360°} \qquad \textbf{[A1]}$$

$$\cos\theta = -\frac{1}{2} \qquad \textbf{[M1]}$$

$$\alpha = 60° \text{ (basic angle)}$$

$$\boldsymbol{\theta = 180° - 60°, 180° + 60° = 120°, 240°} \qquad \textbf{[A1]}$$

$$\boldsymbol{\theta = 0°, 120°, 180°, 240°, 360°}$$

*[Note: Never divide by sin θ: that loses 0°, 180° and 360°. cos θ ≠ 0 at all five answers, so tan θ is defined.]*

*[Note: Marks [4] estimated: the bracket was not legible in our copy.]*

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**6** **[Equation of a curve from its gradient]**

[6]

$k(2)(5(2)-2)=4$ **[M1]**

$16k=4$

$k=\frac{1}{4}$ **[A1]**

$\frac{dy}{dx}=\frac{1}{4}x(5x-2)=\frac{5}{4}x^2-\frac{1}{2}x$

$y=\frac{5}{12}x^3-\frac{1}{4}x^2+c$ **[M1]** **[A1]**

$3=\frac{5}{12}(2)^3-\frac{1}{4}(2)^2+c$ **[M1]**

$3=\frac{10}{3}-1+c$

$c=\frac{2}{3}$

$\boldsymbol{y=\frac{5}{12}x^3-\frac{1}{4}x^2+\frac{2}{3}}$ **[A1]**

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**7** **[Trig identity; hence solve]**

**(a)** [3]

$\text{LHS}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ **[M1]**

$=\frac{1}{\sin\theta\cos\theta}$

$=\frac{2}{2\sin\theta\cos\theta}$ **[M1]**

$=\frac{2}{\sin 2\theta}=\text{RHS (shown)}$ **[A1]**

*[Note: Marks [3] estimated: the bracket was not legible in our copy.]*

**(b)** [3]

$\frac{2}{\sin 2\theta}=4$

$\sin 2\theta=\frac{1}{2}$ **[M1]**

For $0°\le\theta\le 45°$, $0°\le 2\theta\le 90°$, so only the acute value counts:

$2\theta=30°$ **[M1]**

$\boldsymbol{\theta=15°}$ **[A1]**

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**8** **[Optimisation: cylinder in a cone]**

**(a)** [3]

Let the height of the cylinder be $h$ cm. The small cone above the cylinder is similar to the whole cone:

$\frac{r}{4} = \frac{12-h}{12}$ **[M1]**

$h = 12 - 3r$ **[A1]**

$V = \pi r^2 h = \pi r^2(12-3r)$

$= 3\pi(4r^2 - r^3)$ (shown) **[A1]**

**8** *(continued)*

**(b)** [6]

$\frac{dV}{dr} = 3\pi(8r - 3r^2) = 24\pi r - 9\pi r^2$ **[M1]**

$3\pi r(8-3r) = 0$ **[M1]**

$r = \frac{8}{3}$ $(r \neq 0)$ **[A1]**

$V = 3\pi\left[4\left(\frac{8}{3}\right)^2 - \left(\frac{8}{3}\right)^3\right]$ **[M1]**

$= \frac{256\pi}{9}$ **[A1]**

$\approx$ **89.4 cm³ (3 s. f.)** **[A1]**

*[Note: Factorise, do not divide by r: dV/dr = 3πr(8 − 3r). Check: h = 12 − 3(8/3) = 4 cm.]*

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**9** **[Partial fractions]**

[4]

$\frac{11x-3}{(x+2)(3x+1)} = \frac{A}{x+2} + \frac{B}{3x+1}$ **[B1]**

$11x - 3 = A(3x+1) + B(x+2)$ **[M1]**

$x = -2$: $-25 = -5A$

$A = 5$ **[A1]**

$x = -\frac{1}{3}$: $-\frac{20}{3} = \frac{5}{3}B$

$B = -4$ **[A1]**

$$\frac{\mathbf{11x-3}}{\mathbf{(x+2)(3x+1)}} = \frac{\mathbf{5}}{\mathbf{x+2}} - \frac{\mathbf{4}}{\mathbf{3x+1}}$$

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**10** **[Definite integral]**

[4]

$$(1+\sqrt{x})^2 = 1 + 2x^{\frac{1}{2}} + x \qquad \textbf{[M1]}$$

$$\int_1^4 \left(1 + 2x^{\frac{1}{2}} + x\right) dx = \left[x + \frac{4}{3}x^{\frac{3}{2}} + \frac{x^2}{2}\right]_1^4 \qquad \textbf{[M1]}$$

$$= \left(4 + \frac{32}{3} + 8\right) - \left(1 + \frac{4}{3} + \frac{1}{2}\right) \qquad \textbf{[M1]}$$

$$= \mathbf{\frac{119}{6}} \qquad \textbf{[A1]}$$

*[Note: Expand the bracket first: there is no rule that integrates (1 + √x)² directly.]*

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**11** **[Trig graph: sketch, period, amplitude]**

**(a)**

Cosine shape with one maximum, at (0, 2) **[B1]**

Minimum −6 at both ends, (±π/3, −6) **[B1]**

Crosses the x-axis at x = ±π/9 **[B1]**

[3]

**(b)** [2]

**Period = $\frac{2\pi}{3}$** **[B1]** **Amplitude = 4** **[B1]**

*[Note: The −2 shifts the graph down; it does not change the amplitude.]*

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**12** **[Quadratic inequalities]**

[6]

$$x^2 - 9 \leq 0$$
$$(x+3)(x-3) \leq 0 \qquad \textbf{[M1]}$$
$$-3 \leq x \leq 3 \qquad \textbf{[A1]}$$

$$3x^2 + 19x + 20 < 0$$
$$(3x+4)(x+5) < 0 \qquad \textbf{[M1]}$$
$$-5 < x < -\frac{4}{3} \qquad \textbf{[A1]}$$

Both must hold, so take the overlap:

$$\mathbf{-3 \leq x < -\frac{4}{3}} \qquad \textbf{[A1]}$$

Closed circle at −3, open circle at −4/3 **[B1]**

*[Note: "≤ 0" keeps the end point (closed circle); "< 0" does not (open circle).]*

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**13** **[Quadratic equation in context]**

**(a)** [2]

Let the second radius be $r$ cm.

$2\pi x + 2\pi r = 32\pi$ **[M1]**

$x + r = 16$

$r = 16 - x$ (shown) **[A1]**

**(b)** [4]

$\pi x^2 + \pi(16 - x)^2 = 136\pi$ **[M1]**

$x^2 + 256 - 32x + x^2 = 136$

$2x^2 - 32x + 120 = 0$

$x^2 - 16x + 60 = 0$

$(x - 6)(x - 10) = 0$ **[M1]**

$x = 6$ or $x = 10$ **[A1]**

The two radii are 6 cm and 10 cm, so the pieces are:

**$2\pi(6) = 12\pi \approx 37.7$ cm and $2\pi(10) = 20\pi \approx 62.8$ cm** **[A1]**

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